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Theory:

Forced vibration is of two types:
1. Undamped forced vibration
2. Damped forced vibration

Undamped forced vibration :

The differential governing equation for system without damping is
$m\ddot{u} + ku = p_o \sin\omega_t$
where, m = mass; $\ddot{u}$ = acceleration; k = stiffness; u = displacement; ω_n= natural frequency; p_o= amplitude of force.
Solution of the given equation is

$$u(t) = u(0)\cos\omega_n t + \left[\frac{\dot{u}}{\omega_n} - \frac{p_0}{k}\frac{\omega/\omega_n}{1-(\omega/\omega_n)^2}\right] Sin\,\omega_n t + \frac{p_0}{k}\frac{1}{1-(\omega/\omega_n)^2} Sin\,\omega t$$

where,
$\dot{u}(0)$ = Initial Velocity.
$\dot{u}$ = Velocity.

Damped forced vibration: The amplitude of a forced-damped vibration will settle to some value where the energy loss per cycle is exactly balanced by the energy gained.
The governing equation for damped forced vibration is

$$m\ddot{u} + c\dot{u} + ku = p_0 \mathrm{Sin}\,\omega t$$

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = \frac{p_0}{m}\mathrm{Sin}\,\omega t$$

Where,
c = Damping Coefficient.
ξ = Damping Ratio.

The Solution for the given Equation is

$$u(t) = e^{-\xi\omega_n t}\left(A\mathrm{Cos}\,\omega_D t + B\mathrm{Sin}\,\omega_D t\right) + C\mathrm{Sin}\,\omega t + D\mathrm{Cos}\,\omega t$$

Where, ω_D = Damped Frequency.

Where, $$C = \frac{p_0}{k} \frac{1-r^2}{\left(1-r^2\right)^2 + \left(2\xi r\right)^2}$$

$$D = \frac{p_0}{k} \frac{-2\xi r}{\left(1-r^2\right)^2 + \left(2\xi r\right)^2}$$

Also, r = Frequency Ratio.